and is tabulated in Ref. 10. For the adjacent isentropic free jet

$$\rho_{2a}u_{2a} = \frac{P_{01a}}{(T_{01a})^{1/2}}\left[\frac{R}{kg}\frac{2}{k-1}C_{2a}^2(1-C_{2a}^2)^{2/k-1}\right]^{1/2} \quad (14)$$

and from the geometry

$$x = H/\sin\theta_{2a} \quad (15)$$

Combining Eqs. (12, 14, and 15) yields

$$G_d = \frac{HC_{2a}P_{01a}}{\sigma\sin\theta_{2a}}\left(\frac{2kg}{(k-1)RT_{01a}}\right)^{1/2}(1-C_{2a}^2)^{k/k-1}(I_{1,d}-I_{1,j}) \quad (16)$$

Finally, combining Eqs. (5) and (16), one obtains

$$\frac{d}{dt}C_{2a}^2 = \left[\frac{k}{k-1}\frac{\cos\theta_{2a}}{2C_{2a}^2}\left(\frac{2}{k-1}\frac{C_{2a}^2}{1-C_{2a}^2}-1\right)^{1/2}\right]^{-1}\times \left[\frac{\cos\theta_{2a}}{P_{01a}}\left(\frac{dP_{01a}}{dt}-\frac{P_{01a}}{T_{01a}}\frac{dT_{01a}}{dt}\right)-\frac{2C_{2a}}{H\sigma}\left(\frac{2kg\,RT_{01a}}{k-1}\right)^{1/2}(I_{1,d}-I_{1,j})\right] \quad (17)$$

Integration of Eq. (17) for given forcing functions $P_{01a}(t)$ and $T_{01a}(t)$ with $M_{1a}(t)$ used to get θ_{2a} from the steady-state solution, yields $C_{2a}(t)$. Using the isentropic relation for pressure ratio, in terms of Crocco number, finally the base pressure ratio

$$(P_b/P_{01a})(t) = [1 - C_{2a}^2(t)]^{k/k-1}$$

is determined.

Integration of Eq. (17) lends itself to a simple difference solution and has been found to be relatively insensitive to the size of the steps. The last term of Eq. (17) dominates the solution and confirms the tacit assumption that the pumping action of the mixing region plays the major role in the response of the wake to external disturbances. The "restricted theory" has been used since there is a general insufficiency of information on approaching boundary layers and has given good results in the solution of base pressure problems (8).

A quasi-steady theory has been developed that adequately describes the pressure response of a two-dimensional wake to highly transient external flows and that should give approximate results for step changes in the external flow.

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Radiative Heat-Flux Potential

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THE reduction of the integrodifferential equation governing radiative transfer to a pure differential equation has received considerable attention. This is usually accomplished by taking moments of the radiative transport equation (in terms of the specific intensity) for a gray gas in local thermodynamic equilibrium, expanding the specific intensity in spherical harmonics, and keeping only the first term. This assumes that the radiation obeys the Milne-Eddington directional approximation. The system of moment equations is thus closed, and the resulting equation for the first moment (radiative heat flux) is[1, 2]

$$\nabla(\nabla\cdot\mathbf{q}_{\rm rad}) - 3\,\mathbf{q}_{\rm rad} + 4\pi\nabla B = 0 \quad (1)$$

where $\mathbf{q}_{\rm rad}$ is the radiative heat-flux vector, $B = \sigma T^4/\pi$ (σ is the Stefan-Boltzmann constant, and T the temperature), and all distances are measured in terms of optical paths (the absorption coefficient has been absorbed in the ∇). It seems not to have been noticed that Eq. (1) implies

$$\nabla \times \mathbf{q}_{\rm rad} = 0 \quad (2)\dagger$$

Hence, $\mathbf{q}_{\rm rad} = \nabla\psi$, and $\nabla(\nabla^2\psi - 3\psi + 4\pi B) = 0$, or integrating, $\nabla^2\psi - 3\psi + 4\pi B = \text{const}$. The constant of integration may be absorbed in the heat-flux potential ψ, so that

$$\nabla^2\psi - 3\psi = -4\pi B \quad (3)$$

an inhomogeneous Helmholtz equation. The formal solution to Eq. (3) is given by Bateman,[3] and it shows that the radiative heat-flux potential is equal to the integral over-all space of the local emission (B) exponentially attenuated between the parameter point and the argument point in addition to the contributions from the boundary. The divergence of the heat-flux vector equals $3\psi - 4\pi B$, and this bears a remarkable resemblance to the general form for a gray gas in local thermodynamic equilibrium derived by Goulard.[4] However, the difference in the details of the exponential attenuation suggests the difference between the exponential kernel and the exponential integral kernel in the one-dimensional differential approximation.[5]

In conclusion, we have reduced the solution of a vector differential equation with three components [Eq. (1)] to the solution of a single scalar differential equation of standard form [Eq. (3)] by the introduction of the radiative heat-flux

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† By February 10, 1965, the author received word that Eq. (2) and its consequences had been found independently by Traugott.

potential. Moreover, we have noted that the general three-dimensional differential approximation is quite akin to the one-dimensional form in that the Milne-Eddington directional approximation appears to be equivalent to the replacement of an exponential integral by an exponential with a different coefficient.

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Transition of Air Laminar Boundary Layers

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Nomenclature

- w = insulated wall condition
- w_1 = insulated wall condition at end of laminar flow (start of transition to turbulent flow)
- w_2 = insulated wall condition at end of transition (fully developed turbulent flow)
- ∞ = freestream conditions
- s = conditions behind normal shock
- R = universal gas constant
- m = molecular weight
- ρ = density
- u = velocity
- d = length of flow along body
- Re = Reynolds number
- μ = viscosity
- Y = roughness length in boundary layer
- θ^* = boundary-layer momentum thickness
- $\bar{u}$ = fluctuation in freestream velocity (magnitude of velocity disturbance)
- M = characteristic length of freestream turbulence mechanism
- d/M = scale parameter of freestream turbulence
- $\bar{u}/u_\infty$ = magnitude of freestream turbulence
- K_H = Stanton number, dimensionless heat-transfer parameter
- Λ = Karman-Pohlhausen stability parameter
- C_f = skin-friction coefficient
- P = pressure

Introduction

RECENT high enthalpy data indicate rapid increases of transition freestream Reynolds number with increasing enthalpy.[1–4] The boundary layer, however, under these high enthalpy conditions does not see the freestream condition, but another condition, which is much hotter and less dense than the freestream. Figure 1 shows definition of Reynolds numbers for transition.

Von Karman[5] used the insulated adiabatic wall fluid conditions to obtain a modified Reynolds number (Re_w),

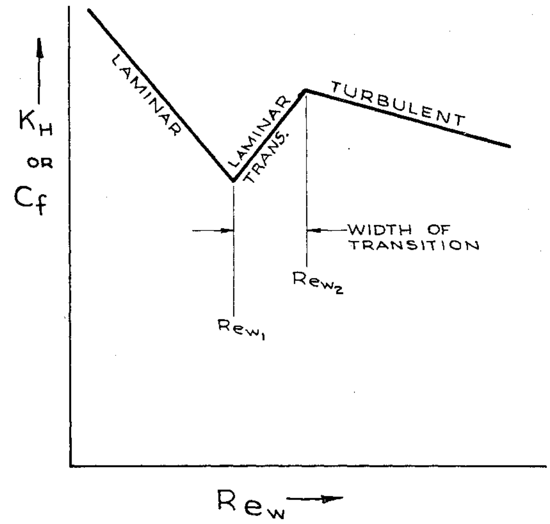

Fig. 1 Definition of Reynolds numbers.

which was then used in order to correct for compressibility effects

$$Re_\infty = \rho_\infty u_\infty d/\mu_\infty$$

is modified to

$$Re_w = \rho_u u_\infty d/\mu_w$$

By use of this early correlation, the data of Refs. 1–4, plus some incompressible data of Refs. 6–8, are correlated independent of velocity in the 450- to 16,000-fps region. This is seen in Fig. 2. A residual parameter, describing the size of freestream or roughness element at the wall relative to the boundary-layer momentum thickness, is present in data at all these velocities. The final Reynolds numbers (Re_{w_1}) for transition obtained vary from 250,000, for a roughness element of 1% of the momentum thickness, to 26,000, for a roughness element equal to the momentum thickness.

Effect of High Enthalpy

In order to transform the freestream Reynolds numbers into local modified Reynolds numbers as described previously, a series of computations was made to change the freestream conditions to conditions behind the normal shock. This was done with the Rankine-Hugoniot condition for mass-flow observation, $\rho_\infty u_\infty = \rho_s u_s$, coupled with the real gas viscosity change due to stagnation temperature rise. This latter rise of the local viscosity is the only change in the freestream Reynolds numbers under these conditions. The Reynolds number behind the normal shock is less, due to this rise in the viscosity

$$Re_s = \rho_s u_s d/\mu_s = \rho_\infty u_\infty d/\mu_s$$

A second series of computations was made to transform the freestream Reynolds numbers into equivalent modified Reynolds numbers as described previously. This series was made by the assumption that the static pressure was conserved across the boundary layer, and hence the following equation held that

$$P_\infty = P_w = \rho_\infty(R/m_\infty)T_\infty = \rho_w(R/m_w)T_w$$

These flat-plate Reynolds numbers (Re_w) are considerably less than the local Reynolds numbers behind the normal shock (Re_s), due to the additional drop of the local density. Both sets of local Reynolds numbers are plotted against velocity in Fig. 2. It will be seen for satellite conditions, 25,000 fps, that the normal-shock local Reynolds numbers are a factor of 7 below the freestream Reynolds numbers, whereas the flat-

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